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SHORT COMMUNICATION

REPLY: 'MACROSCALE SURFACE ROUGHNESS AND FRICTIONAL RESISTANCE IN OVERLAND FLOW'

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ABSTRACT

The model for characterizing frictional resistance in overland flow as a function of surface roughness inundation presented in Lawrence (1997, *Earth Surface Processes and Landforms*, **22**, 365–382) is neither in error nor at odds with physical intuition. The proposed model provides a simple, physically derived explanation for the relationship between frictional resistance and flow depth during the progressive inundation of surface roughness and is applicable to data acquired under a variety of field and experimental conditions. In maintaining these objectives, the model uses a single length scale for characterizing the surface roughness and it is assumed that the size of the roughness elements can be represented by this one measure. This is the only approach that can be physically justified for the range of data considered. Abraham's alternative model for the case of partial inundation (1998, *Earth Surface Processes and Landforms*, this issue) requires that the surface roughness elements be quantified in terms of two distinct length scales. Although this may be feasible within the context of laboratory and some very detailed field studies, it is not compatible with data more widely available. The alternative model Abrahams proposes is actually only useful for the case of circular cylindrical roughness elements and cannot be applied to objects of arbitrary shape as it is based on an implicit assumption requiring the two principal axes in the horizontal plane to be of equivalent lengths. The correct scaling for non-equidimensional shapes is given by $f \sim d/c$ (where c is the horizontal axis parallel to the direction of flow), rather than the scaling $f \sim d/b$ suggested by Abrahams. The generalization of the proposed heuristic model of Lawrence (1997) to a more precise description of surface geometry should, however, be undertaken with caution, as variations in other equally important factors (e.g. the coefficient of drag, the free surface elevation, and the size distribution of the surface roughness elements) also need to be taken into account in developing a more detailed model. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: overland flow hydraulics; roughness elements; dimensional scalings

The compiled data and model presented in Lawrence (1997) illustrate the apparent non-monotonic trend exhibited by very shallow free surface flows when flow resistance is considered as a function of an inundation ratio, defined as the depth of flow scaled by the characteristic roughness of the surface. The inundation ratio is simply a dimensionless depth and allows data from a wide range of field and laboratory studies to be compared directly. This approach is in contrast with the common technique for presenting overland flow hydraulics data, which typically relies on the flow Reynolds number as the principal dimensionless variable. As pointed out in Lawrence (1997), the Reynolds number often functions as a surrogate for changes in the depth of flow, so that if one wishes to compare flow hydraulics on different surfaces, the ratio of flow depth to surface roughness represents a more useful dimensionless group.

In applying this method, one must identify a length scale for the surface roughness. Although it may initially appear preferable to include as much detail as possible in quantifying the surface geometry, this is incompatible with the other approximations and scalings used in estimating the Darcy–Weisbach friction factor. In other words, a single average flow depth, a single mean flow velocity, and a single average water surface slope which is in turn usually estimated from the mean ground surface slope, are all used in calculating the friction factor. As is the case with surface roughness elements, these physical variables exhibit far more complexity than the one quantity used to represent them may suggest; however, a single value is chosen to represent the magnitude of the given variable.

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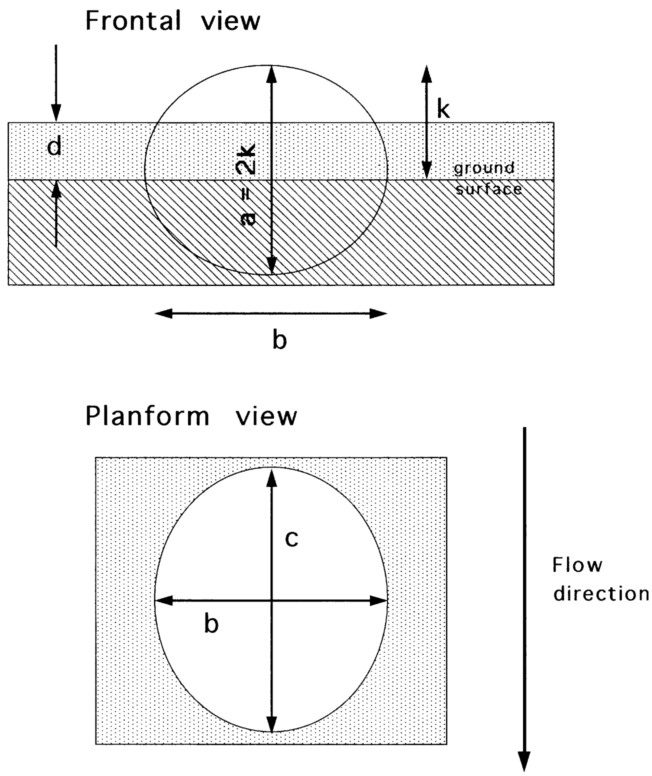

Figure 1. Frontal and planform views of a surface roughness element with a vertical length a, and principal lengths b and c in the horizontal plane. In this case, length b is oriented perpendicular to the flow direction and length c is parallel to it. The height of the roughness element above the ground surface is $k=a/2$, and d is the depth of flow

The use of a single length scale for characterizing the protrusion of surface roughness elements into a flow field is analogous to the use of a single measure of grain size (e.g. D_{50}) studies of sediment entrainment, transport and settling (e.g. Miller *et al.*, 1977; Baba and in Komar, 1981; Wilcock and Southard, 1988; Bridge and Bennett, 1992). Although it is recognized that particle shape may play a critical role in these processes (as does grain size distribution), as a first approximation it is assumed that the geometry of the grains is adequately characterized by a single length scale, thus implicitly assuming an equidimensional shape. Similarly, the model presented in Lawrence (1997) uses a single measure of the surface roughness (corresponding to $D_{50}/2$) as a basis for scaling the depth of flow. This simple scaling allows the physical behaviour of a variety of flows to be compared in a dimensionless form. Most importantly, in the absence of shape data, which are rarely available for surface roughness elements, it is not only the simplest representation of the size of roughness elements, but is the only scaling that can be justified. This scaling is, of course, also directly analogous to the use of D_{50} or D_{90} to characterize surface roughness in gravel-bed rivers (e.g. Bray, 1982).

Abrahams (1998) does not take issue with the general approach proposed, but rather suggests a correction to the model. The proposed correction introduces an additional length scale, b, to account for the breadth of the roughness elements in the horizontal plane perpendicular to the direction of flow (see Figure 1). When this length scale is used, the projected frontal area, A_F, during partial inundation is approximately given by

$$A_F \sim bd \tag{1}$$

where d is the flow depth during partial inundation, rather than by

$$A_F \sim (a/2)\, d \cong kd \tag{2}$$

which corresponds to the case in which the size of the element is specified in terms of a single length scale k,

such that $2k \sim a \cong b \cong c$. However, when estimating the number of roughness elements per unit surface area, n, the planform area of an element, A_{pf}, is given approximately by

$$A_{pf} \sim (\pi/4)\, bc \tag{3}$$

rather than by

$$A_{pf} \sim (\pi/4)\, b^2 \tag{4}$$

as Abrahams proposes. Thus, since the frictional resistance in the flow is

$$f = (8\tau)/(\rho V^2) \tag{5}$$

with

$$\tau = nF_d \tag{6}$$

and

$$F_d \sim C_d (1/2) \rho V^2 A_F \tag{7}$$

the final expression for scaling the frictional resistance is

$$f \sim 4 \underline{P} C_d\, (A_F/A_{pf}) = (16/\pi)\, (d/c)\, \underline{P} C_d \tag{8}$$

where the fractional cover $\underline{P} = nA_{pf}$. Therefore, on a surface with roughness elements which are not equidimensional, the flow resistance scales with d/c, rather than the d/b scaling proposed by Abrahams. His 'error' is introduced by assuming that $b = c$ in the horizontal plane. This assumption would clearly be valid in the case of partially inundated circular cylinders, but would not apply more generally to flow resistance generated by surface roughness elements of other shapes.

Finally, a word of caution is in order concerning the generalization of the proposed model to consider the effects of variable surface geometry at partial inundation. As emphasized throughout the development and presentation of the model in Lawrence (1997), numerous approximations and simplifications are used in developing the scaling arguments which underpin this approach. One of these is the use of a single length scale for characterizing the surface roughness when evaluating flow behaviour at partial inundation. Several other physical factors have also been either simplified or neglected, including the deformation of the free surface during partial to marginal inundation, variations in the coefficient of drag with inundation ratio and with Reynolds number, the potential for wake interference with increasing fractional cover, and the effects of a distribution of sizes of roughness elements, which is of particular relevance to shallow flows over natural rough surfaces. Thus, by expanding the model to incorporate additional length scales for characterizing the geometry of the surface without addressing these issues, one potentially imparts a degree of spurious detail into a method which is otherwise based on order of magnitude estimates.

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